Lösungen FUNKTIONEN IV

1. $a(x) = \underline{\underline{2x^2-2x-4}} = 2(x^2-x-2) = \underline{\underline{2(x-2)(x+1)}}$

$b(x) = \underline{\underline{3x^2-12x+12}} = 3(x^2-4x+4) = \underline{\underline{3(x-2)^2}}$

$c(x) = 4\left(x-\frac{1}{2}\right)^2+1 = 4\left(x^2-x+\frac{1}{4}\right)+1 = 4x^2-4x+1+1$

$= \underline{\underline{4x^2-4x+2}}$, $D = 4^2-4\cdot4\cdot2 = 16-32<0 \Rightarrow$ $\underline{\underline{\text{keine NSF}}}$

$d(x) = \underline{\underline{2x^2-10}} = 2(x^2-5) = \underline{\underline{2(x+\sqrt{5})(x-\sqrt{5})}}$

$(b=0)$

$e(x) = (-2x+5)(2x+1) = \underline{\underline{-4\left(x-\frac{5}{2}\right)\left(x+\frac{1}{2}\right)}}$

$= \underline{\underline{-4x^2+8x+5}}$

$f(x) = \underline{\underline{\frac{1}{3}(x+5)^2}} = \frac{1}{3}(x^2+10x+25) = \underline{\underline{\frac{1}{3}x^2+\frac{10}{3}x+\frac{25}{3}}}$

$g(x) = \underline{\underline{x^2+14x+46}}$, $x_{1/2} = \frac{-14\pm\sqrt{196-184}}{2} = -7\pm2\sqrt{3}$

$\Rightarrow g(x) = \underline{\underline{(x+7-2\sqrt{3})(x+7+2\sqrt{3})}}$

$h(x) = \underline{\underline{2(x+3+3\sqrt{10})(x+3-3\sqrt{10})}} = 2\left((x+3)^2-(3\sqrt{10})^2\right)$

$= 2(x^2+6x+9-90) = \underline{\underline{2x^2+12x-162}}$

$i(x) = \underline{\underline{3x^2+3x-330}} = 3(x^2+x-110) = \underline{\underline{3(x+11)(x-10)}}$

$j(x) = -(x-5)^2+25 = -x^2+10x-25+25$

$= \underline{\underline{-x^2+10x}} = \underline{\underline{-x(x-10)}}$

$k(x) = \frac{1}{27}(x+9)^2+3 = \frac{1}{27}(x^2+18x+81)+3$

$= \underline{\underline{\frac{1}{27}x^2+\frac{2}{3}x+6}} \longrightarrow = 0 \iff \frac{1}{27}(x+9)^2 = -3$

$\iff (x+9)^2 = -81$

$\Rightarrow$ $\underline{\underline{\text{keine NSF}}}$

$l(x) = \underline{\underline{7(x+3)(x-3)}} = 7(x^2-9) = \underline{\underline{7x^2-63}}$

$$m(x) = \frac{1}{13}x^2 - x = \frac{1}{13}x\,(x-13)$$

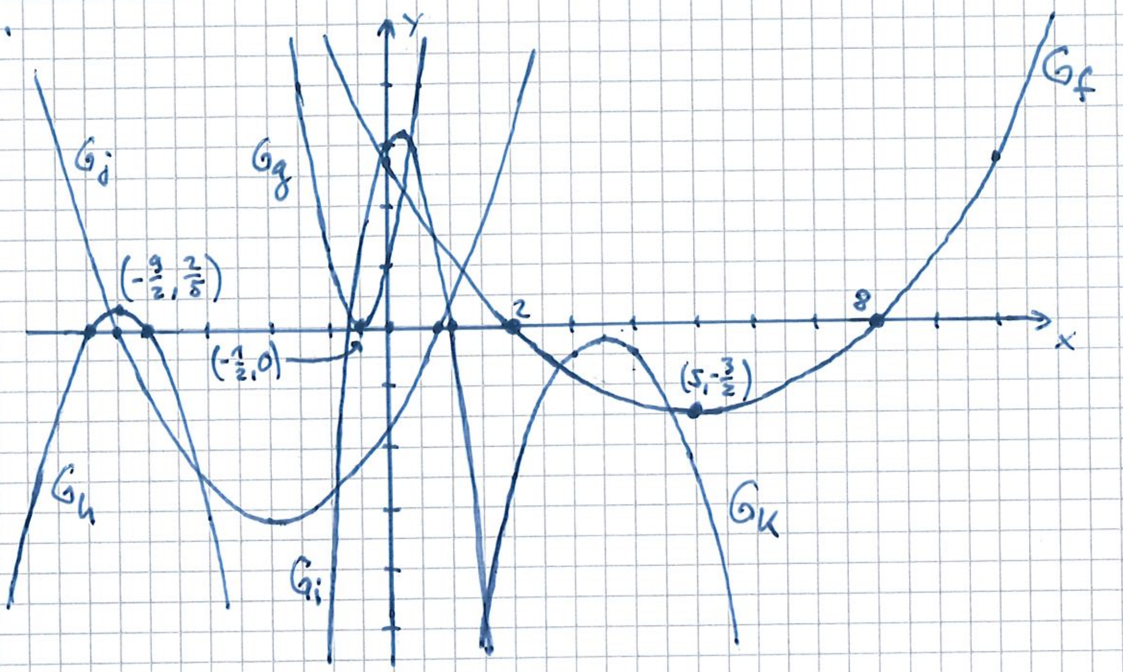
$(c=0)$

$$n(x) = 3\pi x^2 - \pi^2 x - 2\pi^3 = \pi\left(3x^2 - \pi x - 2\pi^2\right)$$

$$= \pi\,(3x+2\pi)(x-\pi)$$

$$o(x) = 6x^2 - 216 = 6(x^2-36) = 6(x+6)(x-6)$$

2.



f: $x_s = \frac{2+8}{2} = 5 \Rightarrow y_s = f(5) = \frac{1}{6}(5-2)(5-8) = \frac{1}{6}\cdot 3\cdot(-3) = -\frac{3}{2}$

$\Rightarrow SP\left(5, -\frac{3}{2}\right)$

g: $g(x) = 4x^2+4x+1 = 4\left(x+\frac{1}{2}\right)^2 \Rightarrow$ SP auf x-Achse $\Rightarrow SP\left(-\frac{1}{2}, 0\right)$

h: $x_s = \frac{-4+(-5)}{2} = -\frac{9}{2} \Rightarrow y_s = h\left(-\frac{9}{2}\right) = -\frac{8}{5}\cdot\left(-\frac{1}{2}\right)\cdot\frac{1}{2} = \frac{2}{5} \Rightarrow SP\left(-\frac{9}{2}, \frac{2}{5}\right)$

i: $i(x) = -5x^2+2x+3 \Rightarrow x_s = \frac{-2}{-10} = \frac{1}{5}$

$\Rightarrow y_s = i\left(\frac{1}{5}\right) = -\frac{5}{25} + \frac{2}{5} + 3 = \frac{16}{5} \Rightarrow SP = \left(\frac{1}{5}, \frac{16}{5}\right)$

j: $j(x) = \frac{1}{12}(2x+9)(3x-2) = \frac{1}{2}\left(x+\frac{9}{2}\right)\left(x-\frac{2}{3}\right) \Rightarrow x_s = \frac{-\frac{9}{2}+\frac{2}{3}}{2} = \frac{\frac{-27+4}{6}}{2} = -\frac{23}{12}$

$\Rightarrow y_s = j\left(-\frac{23}{12}\right) = \frac{1}{12}\left(-\frac{23}{6}+9\right)\left(-\frac{23}{4}-2\right) = \frac{1}{12}\cdot\frac{31}{6}\cdot\frac{-31}{4} = -\frac{962}{288} \approx -3.3$

$\Rightarrow SP\left(-\frac{23}{12}, -3.3\right)$

k: $x_s = \frac{-7}{-2} = \frac{7}{2} \Rightarrow y_s = k\left(\frac{7}{2}\right) = -\frac{49}{4} + \frac{49}{2} - \frac{25}{2}$

$= \frac{49}{4} - \frac{50}{4} = -\frac{1}{4} \Rightarrow SP\left(\frac{7}{2}, -\frac{1}{4}\right)$

3. $f(x) = a\underset{\substack{\uparrow \\ =(x-0)}}{x}(x-5)$ durch $(1,1)$

$\Rightarrow f(1) = a \cdot 1 \cdot (1-5) = -4a \overset{!}{=} 1 \Rightarrow a = -\frac{1}{4}$

$\Rightarrow f(x) = -\frac{1}{4}x(x-5) = \underline{\underline{-\frac{1}{4}x^2 + \frac{5}{4}x}}$

$g(x) = a(x-3)^2$ durch $(1,2)$

$\Rightarrow g(1) = a(1-3)^2 = 4a \overset{!}{=} 2 \Rightarrow a = \frac{1}{2}$

$\Rightarrow g(x) = \frac{1}{2}(x-3)^2 = \frac{1}{2}(x^2-6x+9) = \underline{\underline{\frac{1}{2}x^2 - 3x + \frac{9}{2}}}$

$h(x) = ax^2 + bx - 2$ durch $(1,-1)$ und $(2,-2)$

$\Rightarrow \left| \begin{array}{l} h(1) = -1 \\ h(2) = -2 \end{array} \right| \Rightarrow \left| \begin{array}{l} a+b-2 = -1 \\ 4a+2b-2 = -2 \end{array} \right| \begin{array}{l} (1) \\ (2) \end{array}$

$(2) - 2\cdot(1)$: $2a + 2 = 0 \Leftrightarrow \underline{a = -1}$

in (1): $-1 + b - 2 = -1 \Rightarrow \underline{b = 2}$

$\Rightarrow h(x) = \underline{\underline{-x^2 + 2x - 2}}$

$i(x) = a(x-1)(x-7)$ durch $(3,-1)$

$\Rightarrow a \cdot 2 \cdot (-4) = -8a \overset{!}{=} -1 \Rightarrow a = \frac{1}{8}$

$\Rightarrow i(x) = \frac{1}{8}(x-1)(x-7) = \frac{1}{8}(x^2-8x+7) = \underline{\underline{\frac{1}{8}x^2 - x + \frac{7}{8}}}$

G_j geht durch $(-2,-1)$, $(-1,1)$ und $(1,-2)$

$\left| \begin{array}{l} g(-2) = -1 \\ g(-1) = 1 \\ g(1) = -2 \end{array} \right| \xRightarrow{g(x)=ax^2+bx+c} \left| \begin{array}{l} 4a-2b+c = -1 \\ a-b+c = 1 \\ a+b+c = -2 \end{array} \right| \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$

$(3)-(2)$ $\Rightarrow 2b = -3$ $\Leftrightarrow \underline{b = -\frac{3}{2}}$

$(1)-(2)$: $3a - b = -2 \Rightarrow 3a + \frac{3}{2} = -2 \Leftrightarrow \underline{a = -\frac{7}{6}}$

in (2): $-\frac{7}{6} + \frac{3}{2} + c = 1 \Leftrightarrow c = 1 + \frac{7}{6} - \frac{3}{2} = \frac{6+7-9}{6} = \frac{4}{6} = \underline{\frac{2}{3}}$

$\Rightarrow \underline{\underline{j(x) = -\frac{7}{6}x^2 - \frac{3}{2}x + \frac{2}{3}}}$

4. (a) $f(x) = a\left(x-\frac{3}{2}\right)(x-4)$ mit $f(0)=20$

$\Rightarrow f(0) = a\left(-\frac{3}{2}\right)(-4) = 6a \stackrel{!}{=} 20 \Rightarrow a = \frac{20}{6} = \frac{10}{3}$

$f(x) = \frac{10}{3}\left(x-\frac{3}{2}\right)(x-4) = \frac{10}{3}\left(x^2 - \frac{11}{2}x + 6\right)$

$= \underline{\underline{\frac{10}{3}x^2 - \frac{55}{3}x + 20}}$

$x_S = \frac{\frac{3}{2}+4}{2} = \frac{11}{4} \Rightarrow y_S = f\left(\frac{11}{4}\right) = \frac{10}{3}\left(\frac{11}{4}-\frac{3}{2}\right)\left(\frac{11}{4}-4\right)$

$= \frac{10}{3}\cdot\frac{5}{4}\cdot\frac{-5}{4} = -\frac{125}{24}$

$\Rightarrow \underline{\underline{SP\left(\frac{11}{4}, -\frac{125}{24}\right)}}$

(b) $f(x) = a\left(x-\frac{1}{2}\right)\left(x-\frac{9}{2}\right)$ mit $f(0) = \frac{9}{8}$

$\Rightarrow f(0) = a\left(-\frac{1}{2}\right)\left(-\frac{9}{2}\right) = \frac{9}{4}a \stackrel{!}{=} \frac{9}{8} \Rightarrow \underline{a = \frac{1}{2}}$

$\Rightarrow f(x) = \frac{1}{2}\left(x-\frac{1}{2}\right)\left(x-\frac{9}{2}\right) = \underline{\underline{\frac{1}{2}x^2 - \frac{5}{2}x + \frac{9}{8}}}$

$x_S = \frac{\frac{1}{2}+\frac{9}{2}}{2} = \frac{5}{2} \Rightarrow y_S = f\left(\frac{5}{2}\right) = \frac{1}{2}\left(\frac{5}{2}-\frac{1}{2}\right)\left(\frac{5}{2}-\frac{9}{2}\right)$

$= \frac{1}{2}\cdot 2\cdot(-2) = -2 \Rightarrow \underline{\underline{SP\left(\frac{5}{2}, -2\right)}}$

(c) G_f geht durch den Ursprung $\Rightarrow x=0$ ist NS

$\Rightarrow f(x) = a\underbrace{x}_{=(x-0)}(x+9) \Rightarrow x_S = \frac{0-9}{2} = -\frac{9}{2}$

$\Rightarrow f\left(-\frac{9}{2}\right) = a\cdot\left(-\frac{9}{2}\right)\left(\frac{9}{2}\right) = \frac{81}{4}a \stackrel{!}{=} 6 \Rightarrow a = \frac{24}{81} = \frac{8}{27}$

$\Rightarrow f(x) = \frac{8}{27}x(x+9) = \underline{\underline{\frac{8}{27}x^2 + \frac{8}{3}x}}$ und $\underline{\underline{SP\left(-\frac{9}{2}, 6\right)}}$

(d) $f(x) = ax^2 + bx - 3 \Rightarrow \left| \begin{array}{l} f(-2) = 4a - 2b - 3 \stackrel{!}{=} \frac{1}{2} \\ f(4) = 16a + 4b - 3 \stackrel{!}{=} 8 \end{array} \right| \begin{array}{l} (1) \\ (2) \end{array}$

(2) + 2·(1): $24a - 9 = 9 \Leftrightarrow 24a = 18 \Leftrightarrow \underline{a = \frac{3}{4}}$

in (1): $\cancel{3} - 2b - \cancel{3} = \frac{1}{2} \Leftrightarrow \underline{b = -\frac{1}{4}} \Rightarrow \underline{f(x) = \frac{3}{4}x^2 - \frac{1}{4}x - 3}$

$x_S = \frac{\frac{1}{4}}{2\cdot\frac{3}{4}} = \frac{1}{6} \Rightarrow y_S = f\left(\frac{1}{6}\right) = \frac{3}{4}\cdot\frac{1}{36} - \frac{1}{4}\cdot\frac{1}{6} - 3 = -3\frac{1}{48} \Rightarrow \underline{\underline{SP\left(\frac{1}{6}, -3\frac{1}{48}\right)}}$

5. (a)

$$\left|\begin{array}{l} f(1)=4 \\ f(4)=7 \\ f(7)=1 \end{array}\right| \overset{f(x)=ax^2+bx+c}{\Longrightarrow} \left|\begin{array}{l} a+b+c=4 \\ 16a+4b+c=7 \\ 49a+7b+c=1 \end{array}\right| \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

$$\Rightarrow \begin{array}{l} (3)-(2): \\ (2)-(1): \end{array} \left|\begin{array}{l} 33a+3b=-6 \\ 15a+3b=3 \end{array}\right| \Leftrightarrow \left|\begin{array}{l} 11a+b=-2 \\ -5a-b=-1 \end{array}\right| \begin{array}{l} (4) \\ (5) \end{array}$$

$$\Rightarrow (4)+(5): \quad 6a=-3 \Leftrightarrow \underline{a=-\frac{1}{2}}$$

$$\Rightarrow \text{in } (5): \quad \frac{5}{2}-b=-1 \Leftrightarrow \underline{b=\frac{7}{2}}$$

$$\Rightarrow \text{in } (1): \quad -\frac{1}{2}+\frac{7}{2}+c=4 \Leftrightarrow \underline{c=1}$$

$$\Rightarrow f(x)=\underline{\underline{-\frac{1}{2}x^2+\frac{7}{2}x+1}} = -\frac{1}{2}\left(x^2-7x-2\right)$$

$$\Rightarrow x_{1/2}=\frac{7\pm\sqrt{49+8}}{2}=\frac{7\pm\sqrt{57}}{2}$$

$$\Rightarrow f(x)=\underline{\underline{-\frac{1}{2}\left(x-\frac{7+\sqrt{57}}{2}\right)\left(x-\frac{7-\sqrt{57}}{2}\right)}}$$

(b)

$$\left|\begin{array}{l} f\left(-\frac{5}{2}\right)=3 \\ f\left(\frac{1}{4}\right)=-\frac{9}{8} \\ f(2)=12 \end{array}\right| \overset{f(x)=ax^2+bx+c}{\Longrightarrow} \left|\begin{array}{l} \frac{25}{4}a-\frac{5}{2}b+c=3 \\ \frac{1}{16}a+\frac{1}{4}b+c=-\frac{9}{8} \\ 4a+2b+c=12 \end{array}\right| \begin{array}{l} (1) \\ (2) \\ (3) \end{array}$$

$$\Rightarrow \begin{array}{l} (3)-(2): \\ (3)-(1): \end{array} \left|\begin{array}{l} \frac{63}{16}a+\frac{7}{4}b=\frac{105}{8} \\ -\frac{9}{4}a+\frac{9}{2}b=9 \end{array}\right| \Leftrightarrow \left|\begin{array}{l} \frac{9}{8}a+\frac{1}{2}b=\frac{15}{4} \\ \frac{1}{4}a-\frac{1}{2}b=-1 \end{array}\right| \begin{array}{l} (4) \\ (5) \end{array}$$

$$\Rightarrow (4)+(5) \quad \frac{11}{8}a=-\frac{11}{4} \Leftrightarrow \underline{a=2}$$

$$\Rightarrow \text{in } (5): \quad \frac{1}{2}-\frac{1}{2}b=-1 \Rightarrow \underline{b=3}$$

$$\Rightarrow \text{in } (3): \quad 8+6+c=12 \Rightarrow \underline{c=-2}$$

$$\Rightarrow f(x)=\underline{\underline{2x^2+3x-2}}=(2x-1)(x+2)$$

$$=\underline{\underline{2\left(x-\frac{1}{2}\right)(x+2)}}$$

6. (a) $f(x) = -x^2 - 6x - 8 = -(x^2 + 6x + 8) = -(x+4)(x+2)$

$\Rightarrow$ NS: $x_1 = -4$ und $x_2 = -2$

$x_s = \frac{-4 + (-2)}{2} = -3 \Rightarrow y_s = f(-3) = -9 + 18 - 8 = 1$

$\Rightarrow$ SP$(-3, 1)$

$f\left(\frac{18}{5}\right) = -\frac{324}{25} - \frac{108}{5} - 8 = -\frac{324 + 540 + 200}{25} = -\frac{1064}{25}$

$= -42\frac{14}{25} > -43$

$\Rightarrow$ Damit liegt $P\left(\frac{18}{5}, -43\right)$ unterhalb des G_f, denn der Punkt $Q\left(\frac{18}{5}, -42\frac{14}{25}\right)$ liegt auf dem G_f.

(b) $u(x) = \frac{1}{10}x^2 - 2x + c \Rightarrow x_s = -\frac{b}{2a} = \frac{2}{2 \cdot \frac{1}{10}} = 10$

$\Rightarrow y_s = u(x_s) = \frac{1}{10} \cdot 100 - 2 \cdot 10 + c = 10 - 20 + c = -10 + c$

$\Rightarrow$ SP$(10, -10 + c)$

7. (a) $f(x) = ax^2 + c \Rightarrow$ Parameter $b = 0$

$\Rightarrow$ Parabel geht mit Steigung 0 durch y-Achse

$\Rightarrow$ Der SP muss genau auf der y-Achse liegen!

(b) $f(x) = ax^2 + bx = \underset{\substack{\uparrow \\ = x - 0}}{ax}\left(x + \frac{b}{a}\right)$

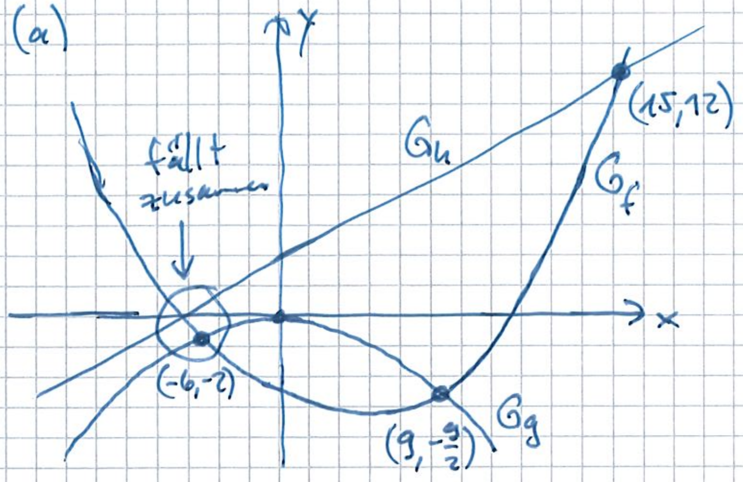
$\Rightarrow x = 0$ ist eine NS $\Rightarrow$ Die Parabel verläuft durch den Ursprung $(0,0)$!

(c) $h(x) = x^2 + bx + c \stackrel{!}{=} (x - x_0)^2$

$\Rightarrow x_0$ ist doppelte NS $\Rightarrow$ SP liegt auf x-Achse!

(Stelle bei $x_s = x_0 = -\frac{b}{2a} = \frac{-b}{2}$.)

8. (a)



$\Rightarrow$ Nicht so klar, ob sich G_g und G_h schneiden...

(b) $f \cap g$: $\frac{5}{36}x^2 - \frac{7}{12}x - \frac{21}{2} = -\frac{1}{18}x^2 \quad | \cdot 36$

$\Leftrightarrow 5x^2 - 21x - 378 = -2x^2 \quad | +2x^2$

$\Leftrightarrow 7x^2 - 21x - 378 = 0 \quad | :7$

$\Leftrightarrow x^2 - 3x - 54 = 0 \quad |$ Zweikl. ansatz

$\Leftrightarrow (x+6)(x-9) = 0$

$\Rightarrow \underline{x_1 = -6} \Rightarrow y_1 = g(-6) = -\frac{1}{18} \cdot 36 = -2 \Rightarrow \underline{\underline{S_1(-6,-2)}}$

$\Rightarrow \underline{x_2 = 9} \Rightarrow y_2 = g(9) = -\frac{1}{18} \cdot 81 = -\frac{9}{2} \Rightarrow \underline{\underline{S_2(9,-\frac{9}{2})}}$

$f \cap h$: $\frac{5}{36}x^2 - \frac{7}{12}x - \frac{21}{2} = \frac{2}{3}x + 2 \quad | -\frac{2}{3}x - 2$

$\Leftrightarrow \frac{5}{36}x^2 - \frac{15}{12}x - \frac{25}{2} = 0 \quad | \cdot \frac{36}{5}$

$\Leftrightarrow x^2 - 9x - 90 = 0$

$\Leftrightarrow (x+6)(x-15) = 0$

$\Rightarrow \underline{x_3 = -6} \Rightarrow S_3 = \underline{\underline{S_1(-6,-2)}}$

$\Rightarrow \underline{x_4 = 15} \Rightarrow y_4 = h(15) = \frac{2}{3} \cdot 15 + 2 = 12$

$\Rightarrow \underline{\underline{S_4(15,12)}}$

$g \cap h$: $-\frac{1}{18}x^2 = \frac{2}{3}x + 2 \quad | -\frac{2}{3}x - 2$

$\Leftrightarrow -\frac{1}{18}x^2 - \frac{2}{3}x - 2 = 0 \quad | \cdot(-18)$

$\Leftrightarrow x^2 + 12x + 36 = 0$

$\Leftrightarrow (x+6)^2 = 0 \Rightarrow x_5 = -6$

$\Rightarrow$ g und h berühren sich $\Rightarrow \underline{\underline{S_5 = S_1(-6,-2)}}$

(c) f: $x_s = \frac{-b}{2a} = \frac{\frac{7}{12}}{2\cdot\frac{5}{36}} = \frac{7}{\cancel{12}_{2}}\cdot\frac{\cancel{18}^{3}}{5} = \frac{21}{10}$

$\Rightarrow y_s = f\left(\frac{21}{10}\right) = \frac{\cancel{5}}{\cancel{36}_{12}}\cdot\frac{\cancel{21}^{7}}{\cancel{10}_{2}}\cdot\frac{21}{10} - \frac{7}{12}\cdot\frac{21}{10} - \frac{21}{2}$

$= \frac{21}{2}\cdot\left(\frac{7}{120} - \frac{7}{60} - 1\right) = \frac{\cancel{21}^{7}}{2}\cdot\frac{7-14-120}{\cancel{120}_{40}}$

$= -\frac{7\cdot 127}{80} = -\frac{889}{80} = -11\frac{9}{80}$

$\Rightarrow \underline{\underline{SP\left(\frac{21}{10} ; -11\frac{9}{80}\right)}}$

g: $x_s = 0 \Rightarrow y_s = 0 \Rightarrow \underline{\underline{SP(0,0)}}$

(d) Keine Lösungen

9. $f(-2) = \frac{5}{54}\cdot 4 + \frac{1}{6}\cdot 2 + \frac{2}{3} = \frac{20+18+36}{54} = \frac{74}{54} = \frac{37}{27}$

$y_P = \frac{4}{3} = \frac{36}{27} < \frac{37}{27} \Rightarrow$ <u>P liegt unterhalb des G_f.</u>

$f\left(\frac{4}{5}\right) = \frac{\cancel{5}}{54}\cdot\frac{4^2}{5^{\cancel{2}}} - \frac{1}{6}\cdot\frac{4}{5} + \frac{2}{3} = \frac{\cancel{16}^{8}}{\cancel{54}_{27}\cdot 5} - \frac{\cancel{4}^{2}}{\cancel{30}_{15}} + \frac{2}{3}$

$= \frac{8 - 18 + 90}{135} = \frac{80}{135}$

$y_Q = \frac{3}{5} = \frac{81}{135} > \frac{80}{135} \Rightarrow$ <u>Q liegt oberhalb des G_f.</u>

$f(6) = \frac{5}{\cancel{54}_{9}}\cdot 6^{\cancel{2}} - \frac{1}{\cancel{6}}\cdot\cancel{6} + \frac{2}{3} = \frac{30 - 9 + 6}{9} = \frac{27}{9} = 3$

$\Rightarrow \underline{\underline{Q(6,3) \in G_f}}$

10. (a) $f(x) = \frac{1}{5}x^2 + bx +$

f

10. (a) $f(x) = \frac{1}{5}x^2 + bx + c$

$$\left| \begin{array}{l} f(-4) = \frac{16}{5} - 4b + c \overset{!}{=} 4 \\ f(6) = \frac{36}{5} + 6b + c \overset{!}{=} 6 \end{array} \right| \iff \left| \begin{array}{l} -4b + c = \frac{4}{5} \\ 6b + c = -\frac{6}{5} \end{array} \right| \begin{array}{l} 1 \\ 2 \end{array}$$

$\Rightarrow$ 2 $-$ 1 : $10b = -\frac{10}{5} \Rightarrow \underline{b = -\frac{1}{5}}$

$\Rightarrow$ in 1 : $\frac{4}{5} + c = \frac{4}{5} \Rightarrow \underline{c = 0}$

$\Rightarrow f(x) = \frac{1}{5}x^2 - \frac{1}{5}x = \underline{\underline{\frac{1}{5}x(x-1)}}$

(b) $f(x) = a\left(x - \frac{4}{3}\right)^2$

$f(0) = a \cdot \left(-\frac{4}{3}\right)^2 = \frac{16}{9}a \overset{!}{=} -3 \Rightarrow \underline{a = -\frac{27}{16}}$

$\Rightarrow f(x) = -\frac{27}{16}\left(x - \frac{4}{3}\right)^2 = -\frac{27}{16}\left(x^2 - \frac{8}{3}x + \frac{16}{9}\right)$

$= \underline{\underline{-\frac{27}{16}x^2 + \frac{9}{2}x - 3}}$

(c) $x_1 = -3$ ist NS mit $x_S = 3 = \frac{x_1 + x_2}{2}$

$\Rightarrow x_2 = 2x_S - x_1 = 6 - (-3) = 9$

$\Rightarrow f(x) = a(x+3)(x-9)$

$\Rightarrow f(1) = a \cdot 4 \cdot (-8) = -32a \overset{!}{=} 2$

$\Rightarrow \underline{a = -\frac{1}{16}}$

$\Rightarrow f(x) = -\frac{1}{16}(x+3)(x-9) = -\frac{1}{16}(x^2 - 6x - 27)$

$= \underline{\underline{-\frac{1}{16}x^2 + \frac{3}{8}x + \frac{27}{16}}}$

11. $f \cap g$: $-\frac{1}{5}x^2 + \frac{12}{5}x - \frac{16}{5} = \frac{1}{2}x + \frac{21}{16} \quad | \cdot 10$

$\Leftrightarrow -2x^2 + 24x \underbrace{-32}_{=-\frac{256}{8}} = 5x + \frac{105}{8} \quad | -5x - \frac{105}{8}$

$\Leftrightarrow -2x^2 + 19x - \frac{361}{8} = 0 \quad | \cdot (-8)$

$\Leftrightarrow 16x^2 - 152x + 361 = 0 \quad |$ bin. Formel

$\Leftrightarrow (4x - 19)^2 = 0 \Rightarrow$ nur 1 Lsg.

$\Rightarrow x_B = \frac{19}{4}$ ist Berührungsstelle

$y_B = g\left(\frac{19}{4}\right) = \frac{1}{2} \cdot \frac{19}{4} + \frac{21}{16} = \frac{38+21}{16} = \frac{59}{16}$

$f \cap h$: $-\frac{1}{5}x^2 + \frac{12}{5}x - \frac{16}{5} = -x^2 + 3 \quad | + x^2 - 3$

$\Leftrightarrow \frac{4}{5}x^2 + \frac{12}{5}x - \frac{31}{5} = 0 \quad | \cdot 5$

$\Leftrightarrow 4x^2 + 12x - 31 = 0$

$\Rightarrow x_{1/2} = \frac{-12 \pm \sqrt{12^2 + 16 \cdot 31}}{8} = \frac{-12 \pm 4\sqrt{3^2 + 31}}{8}$

$= \frac{-3 \pm \sqrt{40}}{2} = -\frac{3}{2} \pm \sqrt{10}$

$\Rightarrow y_1 = h\left(-\frac{3}{2} + \sqrt{10}\right) = -\left(-\frac{3}{2} + \sqrt{10} + \sqrt{3}\right)\left(-\frac{3}{2} + \sqrt{10} - \sqrt{3}\right)$

$= -\left(\frac{9}{4} + 10 - 3 - 3\sqrt{10}\right) = -\frac{37}{4} + 3\sqrt{10}$

$y_2 = h\left(-\frac{3}{2} - \sqrt{10}\right) = -\left(-\frac{3}{2} - \sqrt{10} + \sqrt{3}\right)\left(-\frac{3}{2} - \sqrt{10} - \sqrt{3}\right)$

$= -\left(\frac{9}{4} + 10 - 3 + 3\sqrt{10}\right) = -\frac{37}{4} - 3\sqrt{10}$

$\Rightarrow S_1\left(-\frac{3}{2} + \sqrt{10}, -\frac{37}{4} + 3\sqrt{10}\right)$

$S_2\left(-\frac{3}{2} - \sqrt{10}, -\frac{37}{4} - 3\sqrt{10}\right)$

$f \cap j$: $-\frac{1}{5}x^2 + \frac{12}{5}x - \frac{16}{5} = 2 \quad | -2$

$\Leftrightarrow -\frac{1}{5}x^2 + \frac{12}{5}x - \frac{26}{5} = 0 \quad | \cdot(-5)$

$\Leftrightarrow x^2 - 12x + 26 = 0$

$\Rightarrow x_{1/2} = \frac{12 \pm \sqrt{144 - 104}}{2} = 6 \pm \sqrt{10}$

$\Rightarrow \underline{\underline{S_1(6+\sqrt{10}, 2), \; S_2(6-\sqrt{10}, 2)}}$

$f \cap k$: $-\frac{1}{5}x^2 + \frac{12}{5}x - \frac{16}{5} = \frac{1}{3}x^2 - \frac{4}{3}x + \frac{10}{3} \quad | \cdot 15$

$\Leftrightarrow -3x^2 + 36x - 48 = 5x^2 - 20x + 50 \quad | +3x^2 - 36x + 48$

$\Leftrightarrow 8x^2 - 56x + 98 = 0 \quad | : 2$

$\Leftrightarrow 4x^2 - 28x + 49 = 0$

$\Leftrightarrow (2x - 7)^2 = 0$

$\Rightarrow$ Berührung über $\underline{x_B = \frac{7}{2}}$

$y_B = k\left(\frac{7}{2}\right) = \frac{1}{3} \cdot \frac{49}{4} - \frac{4}{3} \cdot \frac{7}{2} + \frac{10}{3} = \frac{49 - 56 + 40}{12} = \frac{33}{12}$

$\Rightarrow \underline{\underline{B\left(\frac{7}{2}, \frac{33}{12}\right)}}$

$f \cap l$: $-\frac{1}{5}x^2 + \frac{12}{5}x - \frac{16}{5} = \frac{1}{2}x + \frac{5}{2} \quad | \cdot(-10)$

$\Leftrightarrow 2x^2 - 24x + 32 = -5x - 25 \quad | +5x + 25$

$\Leftrightarrow 2x^2 - 19x + 57 = 0$

$D = 361 - 456 < 0$

$\Rightarrow$ keine Schnittpunkte!

$g \cap h$: $\frac{1}{2}x + \frac{21}{16} = -x^2 + 3 \quad \vert + x^2 - 3$

$\iff x^2 + \frac{1}{2}x - \frac{27}{16} = 0 \quad \vert \cdot 4$

$\iff 4x^2 + 2x - \frac{27}{4} = 0$

$\implies x_{1/2} = \frac{-2 \pm \sqrt{4 + 108}}{8} = \frac{-2 \pm 4\sqrt{7}}{8} = -\frac{1}{4} \pm \frac{1}{2}\sqrt{7}$

$\implies y_1 = g\left(-\frac{1}{4} + \frac{1}{2}\sqrt{7}\right) = \frac{1}{2}\left(-\frac{1}{4} + \frac{1}{2}\sqrt{7}\right) + \frac{21}{16}$

$= -\frac{2}{16} + \frac{1}{4}\sqrt{7} + \frac{21}{16} = \frac{19}{16} + \frac{1}{4}\sqrt{7}$

$y_2 = g\left(-\frac{1}{4} - \frac{1}{2}\sqrt{7}\right) = \frac{1}{2}\left(-\frac{1}{4} - \frac{1}{2}\sqrt{7}\right) + \frac{21}{16}$

$= -\frac{2}{16} - \frac{1}{4}\sqrt{7} + \frac{21}{16} = \frac{19}{16} - \frac{1}{4}\sqrt{7}$

$\implies S_1\left(-\frac{1}{4} + \frac{1}{2}\sqrt{7}, \frac{19}{16} + \frac{1}{4}\sqrt{7}\right), S_2\left(-\frac{1}{4} - \frac{1}{2}\sqrt{7}, \frac{19}{16} - \frac{1}{4}\sqrt{7}\right)$

$g \cap j$: $\frac{1}{2}x + \frac{21}{16} = 2 \quad \vert - \frac{21}{16}$

$\iff \frac{1}{2}x = \frac{11}{16} \quad \vert \cdot 2$

$\iff x = \frac{11}{8} \implies S\left(\frac{11}{8}, 2\right)$

$g \cap k$: $\frac{1}{2}x + \frac{21}{16} = \frac{1}{3}x^2 - \frac{4}{3}x + \frac{10}{3} \quad \vert \cdot 6$

$\iff 3x + \frac{63}{8} = 2x^2 - 8x + \underbrace{20}_{= \frac{160}{8}} \quad \vert -3x - \frac{63}{8}$

$\iff 2x^2 - 11x + \frac{97}{8} = 0$

$\implies x_{1/2} = \frac{11 \pm \sqrt{121 - 97}}{4} = \frac{11 \pm \sqrt{24}}{4} = \frac{11}{4} \pm \frac{1}{2}\sqrt{6}$

$\implies y_1 = g\left(\frac{11}{4} + \frac{1}{2}\sqrt{6}\right) = \frac{1}{2}\left(\frac{11}{4} + \frac{1}{2}\sqrt{6}\right) + \frac{21}{16} = \frac{22}{16} + \frac{1}{4}\sqrt{6} + \frac{21}{16} = \frac{43}{16} + \frac{1}{4}\sqrt{6}$

$\implies y_2 = \ldots = \frac{43}{16} - \frac{1}{4}\sqrt{6}$

$\implies S_1\left(\frac{11}{4} + \frac{1}{2}\sqrt{6}, \frac{43}{16} + \frac{1}{4}\sqrt{6}\right), S_2\left(\frac{11}{4} + \frac{1}{2}\sqrt{6}, \frac{43}{16} - \frac{1}{4}\sqrt{6}\right)$

$g \cap l$: $\frac{1}{2}x + \frac{21}{16} = \frac{1}{2}x + \frac{5}{2}$ geht nicht! $g + l$ sind parallel!

$h \cap j$: $-x^2 + 3 = 2 \quad | + x^2$

$\Leftrightarrow x^2 - 1 = 0$

$\Leftrightarrow (x+1)(x-1) = 0$

$\Rightarrow x_{1/2} = \pm 1 \Rightarrow \underline{\underline{S_1(1,2), S_2(-1,2)}}$

$h \cap k$: $-x^2 + 3 = \frac{1}{3}x^2 - \frac{4}{3}x + \frac{10}{3} \quad | + x^2 - 3$

$\Leftrightarrow \frac{4}{3}x^2 - \frac{4}{3}x + \frac{1}{3} = 0 \quad | \cdot 3$

$\Leftrightarrow 4x^2 - 4x + 1 = 0$

$\Leftrightarrow (2x-1)^2 = 0$

$\Rightarrow$ Berührung über $x_B = \frac{1}{2}$

$\Rightarrow y_B = h\left(\frac{1}{2}\right) = -\frac{1}{4} + 3 = \frac{11}{4} \Rightarrow \underline{\underline{B\left(\frac{1}{2}, \frac{11}{4}\right)}}$

$h \cap l$: $-x^2 + 3 = \frac{1}{2}x + \frac{5}{2} \quad | + x^2 - 3$

$\Leftrightarrow x^2 + \frac{1}{2}x - \frac{1}{2} = 0$

$\Leftrightarrow 2x^2 + x + 1 = 0$

$\Leftrightarrow (2x-1)(x+1) = 0$

$\Rightarrow x_1 = \frac{1}{2}$ und $x_2 = -1$

$\Rightarrow y_1 = l\left(\frac{1}{2}\right) = \frac{1}{2} \cdot \frac{1}{2} + \frac{5}{2} = \frac{11}{4} \Rightarrow \underline{\underline{S_1\left(\frac{1}{2}, \frac{11}{4}\right)}}$

$y_2 = l(-1) = \frac{1}{2} \cdot (-1) + \frac{5}{2} = \frac{4}{2} = 2 \Rightarrow \underline{\underline{S_2(-1,2)}}$

$g \cap k$: $\frac{1}{3}x^2 - \frac{4}{3}x + \frac{10}{3} = 2 \quad | -2$

$\iff \frac{1}{3}x^2 - \frac{4}{3}x + \frac{4}{3} = 0 \quad | \cdot 3$

$\iff x^2 - 4x + 4 = 0$

$\iff (x-2)^2 = 0$

$\implies$ Berührung über $x_B = 2 \implies \underline{\underline{B(2,2)}}$

$g \cap l$: $2 = \frac{1}{2}x + \frac{5}{2} \quad | -\frac{5}{2}$

$\iff -\frac{1}{2} = \frac{1}{2}x$

$\iff \underline{x = -1} \implies \underline{\underline{S(-1,2)}}$

$k \cap l$: $\frac{1}{3}x^2 - \frac{4}{3}x + \frac{10}{3} = \frac{1}{2}x + \frac{5}{2} \quad | \cdot 6$

$\iff 2x^2 - 8x + 20 = 3x + 15 \quad | -3x - 15$

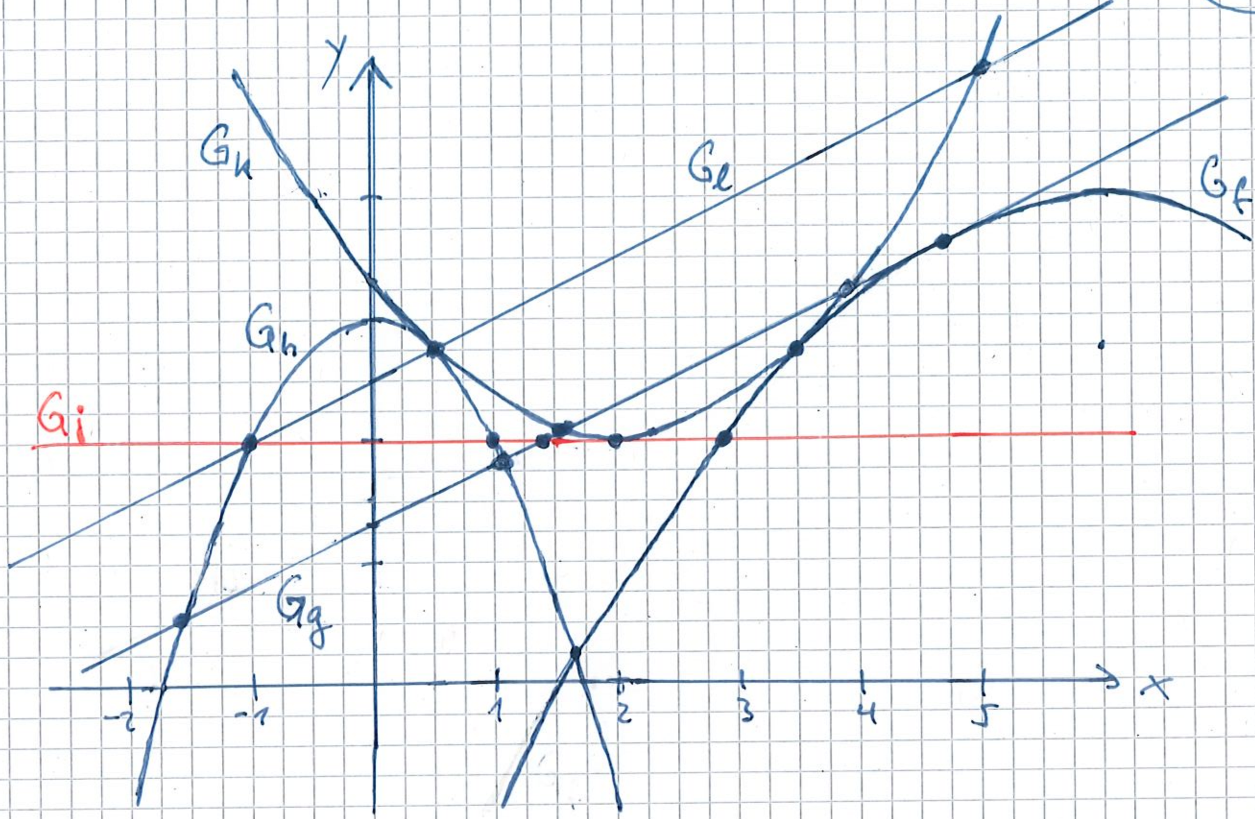
$\iff 2x^2 - 11x + 5 = 0$

$\iff (2x-1)(x-5) = 0$

$\implies x_1 = \frac{1}{2} \quad x_2 = 5$

$\implies y_1 = l\left(\frac{1}{2}\right) = \frac{1}{2} \cdot \frac{1}{2} + \frac{5}{2} = \frac{11}{4} \implies \underline{\underline{S_1\left(\frac{1}{2}, \frac{11}{4}\right)}}$

$y_2 = l(5) = \frac{1}{2} \cdot 5 + \frac{5}{2} = 5 \implies \underline{\underline{S_2(5,5)}}$



12. (a) $-2x^2+3x+1 = mx+3 \quad | -mx-3$

$\iff -2x^2+(3-m)x-2=0$

$\implies D=(3-m)^2-16 \overset{!}{=} 0$

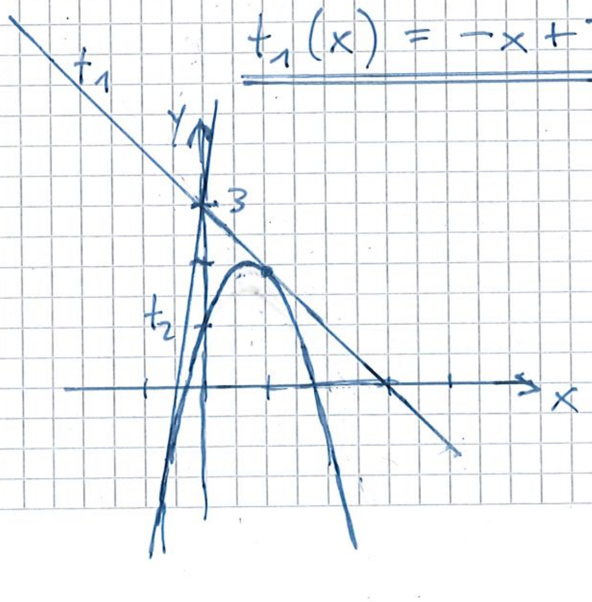
$\iff 9-6m+m^2-16 = 0$

$\iff m^2-6m-7=0$

$\iff (m+1)(m-7)=0$

$\implies$ Es kommen 2 Geraden in Frage:

$\underline{\underline{t_1(x) = -x+3}}$ und $\underline{\underline{t_2(x) = 7x+3}}$



12. (b) $ax^2-2x+5 = -3x+\frac{21}{4} \quad \left| +3x-\frac{21}{4}\right.$

$\Leftrightarrow ax^2+x-\frac{1}{4} = 0$

$\Rightarrow D = 1+a \overset{!}{=} 0 \quad \Rightarrow \underline{a=-1}$

$\Rightarrow -x^2-2x+5 = -3x+\frac{21}{4} \quad \left| +3x-\frac{21}{4}\right.$

$\Leftrightarrow -x^2+x-\frac{1}{4}=0 \quad |\cdot(-4)$

$\Leftrightarrow 4x^2-4x+1=0$

$\Leftrightarrow (2x-1)^2=0 \quad \Rightarrow \underline{x_B=\frac{1}{2}}$

$\Rightarrow y_B = -3\cdot\frac{1}{2}+\frac{21}{4} = -\frac{6}{4}+\frac{21}{4} = \frac{15}{4} \Rightarrow \underline{\underline{B\left(\frac{1}{2}, \frac{15}{4}\right)}}$

(c) Gg ist eine Horizontale! ($g(x)=s=$ konstant)

$\Rightarrow$ Berührungspunkt muss Scheitelpunkt von f sein!

$\Rightarrow x_s = \frac{-b}{2a} = \frac{-8}{2} = -4$

$\Rightarrow y_s = 16+32+10 = \underline{\underline{58 = s}}$

(d) $-2x+3 = a(x-3)^2 = ax^2-6ax+9a+1 \quad |+2x-3$

$\Leftrightarrow \underbrace{a}_{=a}x^2+\underbrace{(2-6a)}_{=b}x+\underbrace{9a-2}_{=c}=0$

$\Rightarrow D = (2-6a)^2-4\cdot a\cdot(9a-2)$

$= 4-24a+\cancel{36a^2}-\cancel{36a^2}+8a$

$= 4-16a \overset{!}{=} 0 \quad \Rightarrow \underline{a=\frac{1}{4}}$

$\Rightarrow -2x+3 = \frac{1}{4}x^2-\frac{6}{4}x+\frac{9}{4}+1 \quad |\cdot 4$

$\Leftrightarrow -8x+12 = x^2-6x+9+4 \quad |+8x-12$

$\Leftrightarrow x^2+2x+1=0$

$\Leftrightarrow (x+1)^2=0 \quad \Rightarrow \underline{x_B=-1}$

$\Rightarrow y_B = -2\cdot(-1)+3 = 5 \Rightarrow \underline{\underline{B(-1,5)}}$

(e) $\frac{1}{2}x^2 + bx + 1 = 2x - \frac{7}{2} \quad | -2x + \frac{7}{2}$

$\Leftrightarrow \frac{1}{2}x^2 + (b-2)x + \frac{9}{2} = 0$

$\Rightarrow D = (b-2)^2 - 4 \cdot \frac{1}{2} \cdot \frac{9}{2} = b^2 - 4b + 4 - 9$

$= b^2 - 4b - 5 \overset{!}{=} 0$

$\Leftrightarrow (b+1)(b-5) = 0$

$\Rightarrow$ 2 Lösungen: $\underline{\underline{b_1 = -1 \text{ und } b_2 = 5}}$

(f) $g: 2x - y = -2 \Leftrightarrow y = 2x + 2$

$\Rightarrow$ Tangente parallel zu g: $t(x) = 2x + q$

$\Rightarrow 5x^2 - 3x + 2 = 2x + q \quad | -2x - q$

$\Leftrightarrow 5x^2 - 5x + 2 - q = 0$

$\Rightarrow D = 25 - 20(2-q)$

$= 25 - 40 + 20q$

$= -15 + 20q \overset{!}{=} 0 \Leftrightarrow \underline{q = \frac{3}{4}}$

$\Rightarrow 5x^2 - 3x + 2 = 2x + \frac{3}{4} \quad | \cdot 4$

$\Leftrightarrow 20x^2 - 12x + 8 = 8x + 3 \quad | -8x - 3$

$\Leftrightarrow 20x^2 - 20x + 5 = 0 \quad | : 5$

$\Leftrightarrow 4x^2 - 4x + 1 = 0$

$\Leftrightarrow (2x-1)^2 = 0 \Rightarrow \underline{x_B = \frac{1}{2}}$

$\Rightarrow y_B = 2 \cdot \frac{1}{2} + \frac{3}{4} = \frac{7}{4} \Rightarrow \underline{\underline{B\left(\frac{1}{2}, \frac{7}{4}\right)}}$